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Scale-specific metrics for adaptive generalization and geomorphic classification of stream features

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Abstract: The Richardson plot has been used to illustrate fractal dimension of naturally occurring landscape features that are sensitive to changes in scale or resolution, such as coastlines and river channels. The Richardson method estimates the length of a path by traversing (i.e., "walking") the path with a specific stride length. Fractal dimension is determined as the slope of the Richardson plot, which shows path length over a range of stride lengths graphed on log-log axes. This paper describes a variant of the Richardson plot referred to as the Scale-Specific Sinuosity (S^3) plot. S^3 is defined as negative one times the slope of the Richardson plot for a given stride length. A plot of S^3 against stride length offers a frequency distribution whose area under the curve reflects total sinuosity, and whose points mark the amount of sinuosity contributed to the total sinuosity at each stride length. Mathematical relations of S^3 with fractal dimension and sinuosity for linear features are described. The S^3 metric is demonstrated and discussed for several linear stream features distributed over the conterminous United States. The S^3 metric can help guide the preservation of stream feature sinuosity during cartographic generalization and may assist automated geomorphic classification of river systems.

Keywords: hydrography, geomorphologic classification, National Hydrography Dataset, sinuosity, adaptive generalization

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1. Introduction

A skilled geomorphologist or earth scientist can infer terrain and geologic conditions from the pattern and content of hydrographic features on a map. Geomorphologic classifications of river systems often include metrics for river sinuosity, meander size and meander frequency (Rosgen 1994; Buffington and Montgomery, 2013). In order to preserve useful geomorphic patterns in generalized hydrographic data, generalization techniques should retain an appropriate portion of river system meander patterns. The Richardson plot (Richardson, 1961) is commonly used to examine the fractal dimension of linear features, such as streams or coastlines (Mandelbrot, 1967). Several authors have searched for relationships between sinuosity and fractal dimension (e.g., Snow, 1989; Andrle, 1992; Klinkenberg, 1992; Montgomery, 1996; Troutman and Karlinger, 1998) with limited success, but they do not quantify the scale-specific nature of these metrics. This paper contributes a derivative of the Richardson plot that enables visual and quantitative analysis of meander patterns in linear stream features across localized scale jumps. The derived plot makes evident a systematic relationship that could guide the automatic classification of stream features to assist geomorphic classification and hydrographic feature generalization.

Beginning with vector representations of linear stream features, Richardson plot points are determined by measuring the length of a stream polyline feature using a series of segment or stride lengths progressing from smaller to larger length. As the stride length increases, smaller details along the line feature are excluded and the total measured length is reduced from its original size. In essence, each point on the Richardson plot measures the length of a simplified version of the original stream, with simplification parameterized by the stride length. A log-log plot is used by convention to display these data, and the slope of this plot is considered an indication of the overall fractal dimension of the stream feature.

A Richardson plot for four large linear stream features extracted from 1:24,000-scale National Hydrography Dataset (NHD) flowline features is shown in Figure 1 using the typical representation with values reported on a logarithmic scale. Although the Richardson plot is typically used to determine a single fractal dimension value, it has been observed that an evaluation of the slope between consecutive pairs of Richardson plot points reveals a pattern related to the frequency of large- and small-scale details, or meanders, in the stream features. This idea is developed into a metric we refer to as scale-specific sinuosity (S^3). Snow (1989) partially alluded to this idea by subdividing the Richardson plot into three possible sections: the upper and lower sections that are asymptotic to near zero slope, and the section in between. He further suggests zero-slope sections represent a "scale of view" where meander details cannot be resolved versus the center section where sinuosity can be measured.

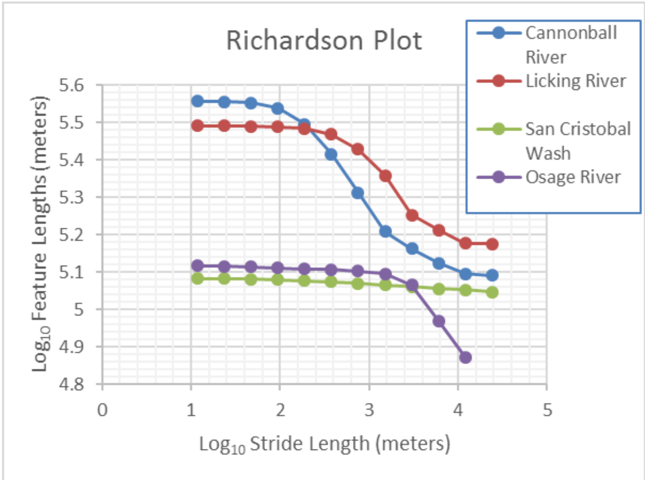

Figure 1. Richardson plot of four linear stream features extracted from 1:24,000-scale National Hydrography Dataset flowline features. Features include the Cannonball River, Licking River, San Cristobal Wash, and the Osage River.

2. Methodology

The S^3 metric and corresponding graph are designed to show line complexity at different resolutions or scales of analysis. The graph is interpretable as a frequency distribution, such that the height of the graph shows the amount of detail at a given scale, and the area under the graph shows the detail in the line. Specifically, the average height of the graph is essentially equal to the fractal dimension minus one, while the total area under the graph is equal to the log of sinuosity.

2.1 Derivation

2.1.1 *Feature Length vs. Stride Length*

Richardson plot computations follow methods described by Bernhardt (1992). The Richardson method estimates the length of a path by walking the path given a specific stride length, i.e. "walking the dividers" (Andrle, 1992; Bernhardt, 1992). For example, walking the path of a sample stream feature is demonstrated in Figure 2. Stride length is referred to as *s*. Eight full strides exist for the sample feature in Figure 2. The length of the feature (F) that remains beyond the eight full

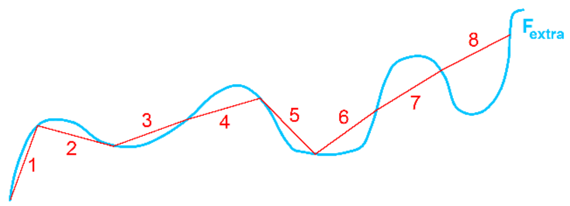

Figure 2. Richardson plot measurements. Set of eight equal-length strides (red) used to estimate "walk" length of stream feature (blue). Left over length of stream beyond the eight strides represented as F_{extra}.

strides in Figure 2 is F_{extra}. Bernhardt (1992) defined an estimate, r_s, of this remaining feature part as an appropriate portion of the stride length *s*, computed as follows:

$$r_s = F_{extra} \times [(n \times s)/(F - F_{extra})], \quad (1)$$

where *F* is the length of the stream feature and *n* is the number of full strides. For the depiction in Figure 2, *F* is equal to the total length of the blue line representing the stream feature, and *n* is eight for the eight equal-length strides of length *s*. The walk length (ℓ) for the feature based on stride length *s* is computed as

$$\ell = n \times s + r_s \quad (2)$$

Applying Bernhardt's (1992) refinement of Richardson's method as described above, the length of each feature is estimated for a series of increasing stride lengths $s_1, s_2, \ldots, s_n$. This results in a set of corresponding walk lengths $\ell_1, \ell_2, \ldots, \ell_n$. The S^3 quantifies the relationship between stride length and feature length. This relationship for the Cannonball River is shown in Figure 3a. We interpret stride length as an indicator of scale/resolution, because for a given stride length, s_i, the method essentially simplifies the feature to remove details smaller than s_i, and then computes the walk length, ℓ_i, of the simplified feature.

2.1.2 *Richardson Plot*

Figure 3 illustrates derivation of the Scale-Specific Sinuosity (S^3) metric. The Richardson plot is a log-

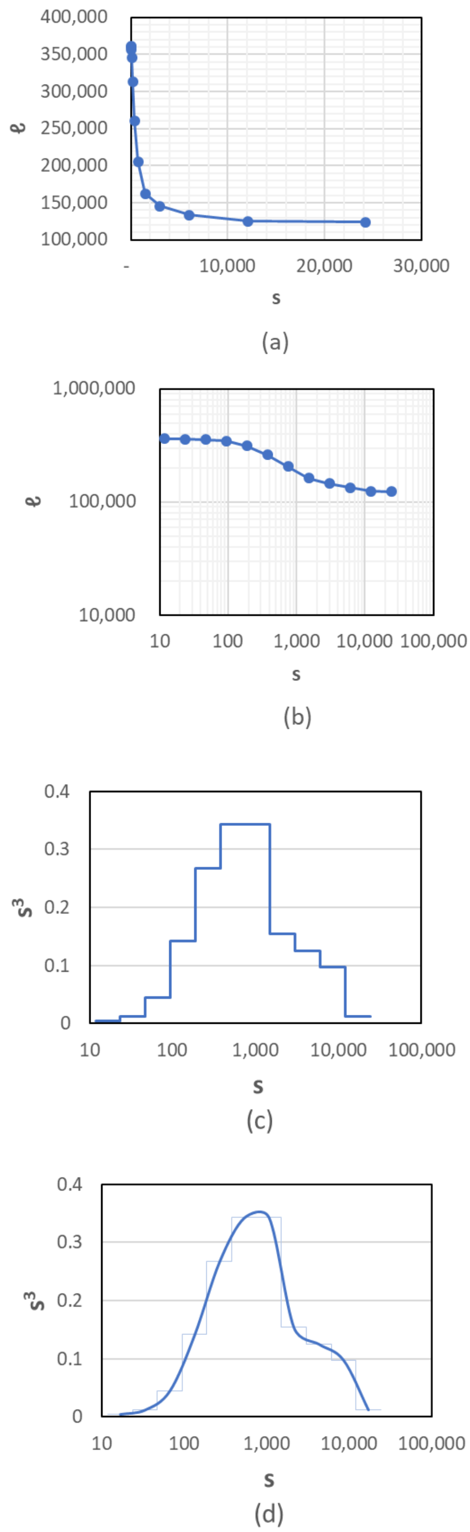

Figure 3: Derivation of scale-specific sinuosity metric (S^3) for a single feature (Cannonball River in Figure 1). (a) Estimated path (or walk) length $\boldsymbol{\ell}$ vs. stride length $\boldsymbol{s}$; (b) same data on a log-log plot, i.e. Richardson plot without the regression line, (c) scale-specific sinuosity (S^3) plot, defined as negative one times the slope of the Richardson plot plotted against stride length on a log scale, (d) continuous version of the S^3 plot.

log plot of feature lengths vs. stride lengths (Figure 3b). The overall slope of the linear regression in a Richardson plot has been widely used to calculate the fractal dimension of a feature (Goodchild, 1980; Mandelbrot, 1982). Specifically, the fractal dimension is calculated as one plus the absolute value of the slope of the best-fit linear regression line through the points of the Richardson plot.

Bernhardt (1992) mentions that a single fractal dimension value may not capture the true character of a line, especially if the R^2 value of the regression line is low. That is, for some line features or across some scale ranges the Richardson plot will exhibit a non-linear or random form. In such cases, the complexity and fractal character of the line cannot be captured in a single metric, as the line exhibits different characters at different scales. Goodchild (1980) argues that landscape features are generated by scale-dependent geomorphic processes that provide clues about the scale of a particular visual representation. Scale-dependent clues are not available in a line or a manifold that is replicated perfectly at every resolution. Therefore, a single fractal dimension value can only apply to a fixed range of scales.

2.1.3 *Scale-Specific Sinuosity (S^3)*

To capture the complexity of a line feature at different scales, we define the *scale-specific sinuosity* (S_x^3) for a given stride length s_x as negative one times the slope of the Richardson plot at a particular location s_x on the x-axis. Given that the Richardson plot is logarithmic, the slope is defined by the logs of ℓ and s. Note that for a finite number of stride lengths, this slope is constant between two consecutive stride lengths. Specifically, for any location s_x between two consecutive stride lengths s_A and s_B, the scale-specific sinuosity can be calculated as:

$$S_x^3 = S_{[A,B]}^3 = -1 \times \frac{\log(\ell_B)-\log(\ell_A)}{\log(s_B)-\log(s_A)} = \frac{\log(\ell_A)-\log(\ell_B)}{\log(s_B)-\log(s_A)} = \frac{log(\ell_A/\ell_B)}{log(s_B/s_A)} \quad (3)$$

This may be interpreted as the ratio of the growth rate of the line feature to the rate of increase in resolution, i.e. the rate at which feature length increases with finer measurement resolution. As with the Richardson plot, we define an S^3 plot by plotting computed S^3 values against stride length s_x, setting the latter on a logarithmic scale (Figure 3c). The concept can be extended to a continuous S^3 curve by imagining an infinite number of stride lengths, or equivalently, considering the limit of S_x^3 as the difference between stride lengths approaches zero (Figure 3d).

2.2 Relations to Existing Metrics

2.2.1 *Relation to Fractal Dimension*

Whereas the S^3 values represent the slopes of the Richardson plot at individual points, the fractal dimension of a line feature is commonly calculated as one plus the absolute slope of the linear regression line through *all* points on the Richardson plot (Mandelbrot, 1967; Bernhardt 1992). Here it is postulated that the fractal dimension of a line feature is simply equal to one plus the average of its S^3 values. Strictly speaking, however, the slope of the overall regression line is not necessarily equal to the average of the slopes at each point. On the other hand, the fractal dimension is a conceptual measure that only has meaning if the regression coefficient is high (Andrle, 1992; Bernhardt 1992). If the points of the Richardson plot all fall on a line then it is self-evident that the slope at each point will equal the slope of the entire line, and therefore the individual S^3 values will equal the overall fractal dimension minus 1. Thus, the S^3 metric can be interpreted as a scale-specific estimate of fractal dimension, essentially addressing the critique that fractal dimension values for a single landscape feature might vary in reflecting geomorphic processes that vary with scale.

2.2.2 *Relation to Sinuosity*

This section relates sinuosity to the total area under the S^3 plot. The total complexity of a line is often expressed using the following measure of sinuosity:

$$sinuosity = \frac{total\ feature\ length}{straight\ line\ distance\ between\ endpoints} \quad (4)$$

Given that the Richardson plot is complete, in other words that the total feature length is captured at the smallest stride length s_1, and the straight line distance between endpoints is captured at the largest stride length s_n, it follows that the standard measure of sinuosity is equal to:

$$sinuosity = \frac{\ell_1}{\ell_n} \quad (5)$$

This measure of sinuosity has a minimum value of 1, but we would like to express the "amount" of complexity in such a way that a perfectly straight line has zero complexity. For this purpose, we compute the log of sinuosity. Noting that $\log(A/B) = \log(A) - \log(B)$, the following expression is derived:

$$\log(sinuosity) = \log(\ell_1) - \log(\ell_n) \quad (6)$$

It is now shown that equation 6 is equivalent to the total area under the S³ curve, as depicted graphically in the plots in Figures 3c and 3d. In Figure 3c, consider each vertical "bar" (defined as a section of constant slope between two measured stride lengths A and B). The area of this bar (denoted $Area_{[A,B]}$) is calculated as the height times the width on the graph. The height of the bar is the value of $S^3_{A,B}$ defined in equation 3, while the width is simply the difference in the x-axis values, i.e. the difference between the logs of the stride lengths. This leads to the following calculation of the area of each bar:

$$Area_{[A,B]} = \frac{\log(\ell_A) - \log(\ell_B)}{\log(s_B) - \log(s_A)} \times [\log(s_B) - \log(s_A)] \quad (7)$$

which reduces to:

$$Area_{[A,B]} = \log(\ell_A) - \log(\ell_B) \quad (8)$$

To calculate the total area under the curve, we simply add up the areas of all bars for stride lengths 1 to n:

$$Area_{[1,n]} = [\log(\ell_1) - \log(\ell_2)] + [\log(\ell_2) - \log(\ell_3)] + \cdots + [\log(\ell_{n-1}) - \log(\ell_n)] \quad (9)$$

The middle terms cancel, yielding:

$$Area_{[1,n]} = \log(\ell_1) - \log(\ell_n) \quad (10)$$

This is exactly equal to the log of sinuosity, as noted in equation 6 above. The analysis can be extended to a continuous curve with equivalent results. Thus, the S³ curve can be interpreted as a log frequency distribution where the area under the curve represents the total log of sinuosity, and the S³ value at each point represents the contribution to total log of sinuosity at each stride length.

Note that the above analysis implies a relationship between sinuosity and fractal dimension, namely:

$$\text{fractal dimension} - 1 = \frac{\log(sinuosity)}{\log(ratio\ of\ measurement\ scales)} \quad (11)$$

The nature of this relationship and its implications warrant further analysis.

3. Illustration with Hydrographic Data

Figure 4 shows the S³ plot for the four stream features whose Richardson plot is shown in Figure 1. Values on the x-axis are stride lengths, but placed on a logarithmic scale, while values on the y-axis are corresponding negative slopes on the Richardson plot. Because the Richardson plot shows both stride lengths and feature lengths on a logarithmic scale, a constant interval on the graph represents a constant rate of increase or decrease. Thus, the slope between consecutive Richardson plot points (y-axis on the S³ plot) indicates the multiplicative factor by which detail is lost relative to a specific coarsening of resolution. We interpret

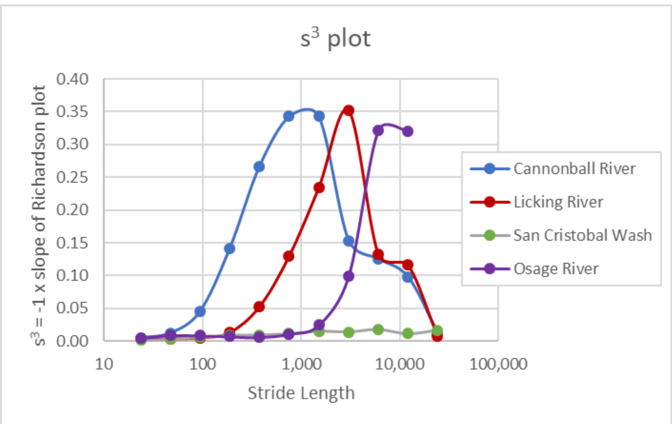

Figure 4: Scale-specific sinuosity (S³) plot. The order of the features (top to bottom in the legend) mirrors the sequence shown in Figure 1 above.

this as a measure of the detail contained in the original feature at that resolution.

The graph has several properties beneficial to adaptive generalization and to geomorphological classification. First, the peaks on the plot corresponds to bend resolutions at which the greatest amount of detail is present in the original stream features. One can readily distinguish differences in large and small meander frequencies among the four sample features, whose plots in Figure 4 are more visually distinct than in the original Richardson plot (Figure 1). Specifically, one can detect graph stride lengths at which meanders of specific stride lengths are eliminated from the line as it undergoes simplification. The four sample lines are illustrated in Figure 5, which separately maps the features, and reports each features unique NatureServe landscape division (NS Division, Comer et al., 2003), sinuosity (ratio of feature length divided by distance between endpoints), point density, and slope range within the respective landscapes. The table contained in each panel of the figure inventories sediment materials underlying the associated feature.

Figure 5a shows that feature sample 3 (the Cannonball River, sinuosity 2.84) carries a large proportion of very small channel bends (0.05 to 1 kilometer), as well as a few larger meanders in the central and eastern portions. The S^3 plot (Figure 4) for the Cannonball River peaks at 1 kilometer. Figure 5b shows feature sample 21 (the Licking River, sinuosity 2.07) in which most bends are about 1 to 5 kilometers wide, and slightly larger than the Cannonball River. The S^3 plot (Figure 4) for the Licking River peaks at about 3 kilometers. In Figure 5d, feature sample 48 (the Osage River, sinuosity 1.83) shows very few small bends but includes several large meanders that are about 2 to 15 kilometers wide. The S^3 plot (Figure 4) for the Osage River peaks at about 10 kilometers. In Figure 5c, feature sample 34 (the San Cristobal Wash, sinuosity 1.09) shows almost no meanders at all, relative to the other four samples. The S^3 plot (Figure 4) for the San Cristobal Wash does not show

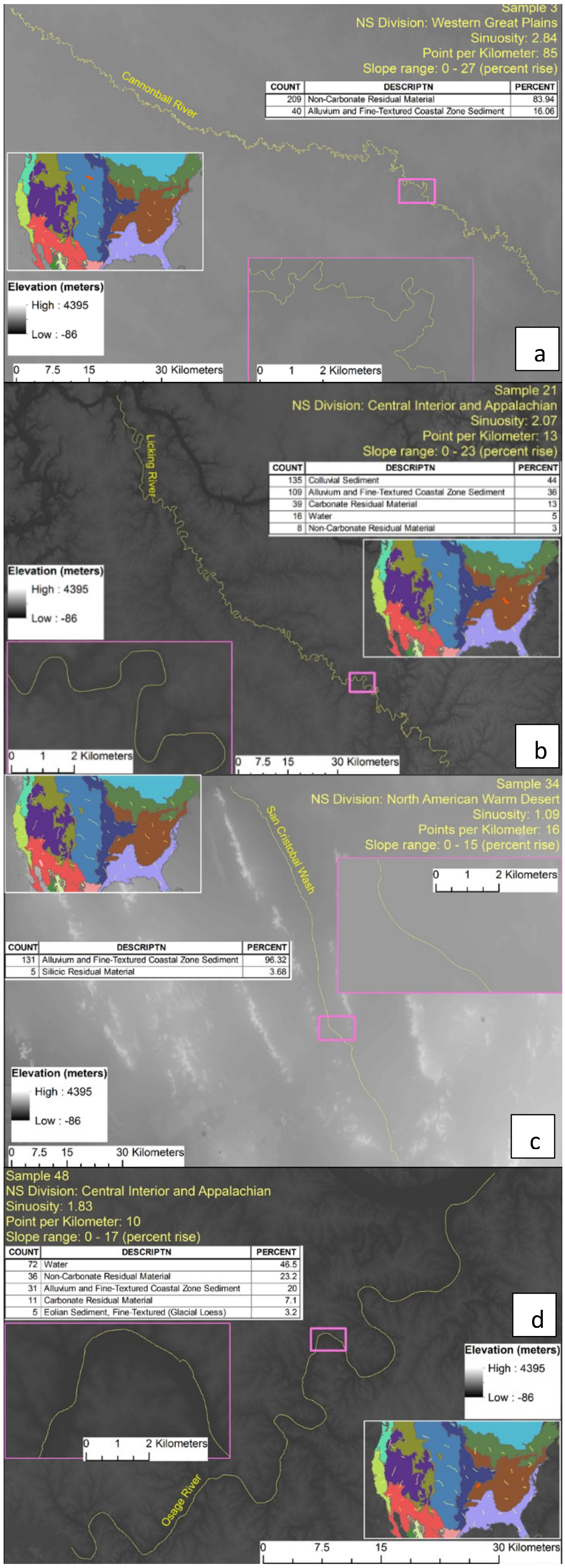

Figure 5. Four sample streams from 1:24,000-scale NHD data and associated feature details including ecologic division, sinuosity, vertex density, elevation slope range, and underlying lithology. Respectively shown from top to bottom panels are (a) Cannonball River, (b) Licking River, (c) San Cristobal Wash, and (d) Osage River. Inset shows enlargement of corresponding portion of feature highlighted in the maroon box.

any substantive peaks relative to the other features.

Second, the variant on the Richardson plot developed here provides a scale-specific progression of sinuosity values, giving more information than the single (global) metric for the entire line sample. Slope between pairs of points in a Richardson plot indicates the rate at which details are lost, with smaller details being eliminated initially. As reported above, for example, the (global) sinuosity values for the four samples range from 1.09 for the San Cristobal Wash (Figure 5c) to 2.84 for the Cannonball River (Figure 5a). The log of these values corresponds to the total area under each curve in Figure 4. The range in y-values indicates that some ranges of stride length do not affect length changes much, while other stride length ranges manifest dramatic changes to feature length overall. These areas reflect localized scale-specific ranges within which the line feature is quite sensitive to small variations in stride length. It is proposed that Scale-Specific Sinuosity (S^3), as shown in the graphs, essentially reports a measure of sinuosity that is local within statistical "scale space". This may be considered analogous to statistics localized in "mapped space", such as the LISA (Local Indicator of Spatial Association) metric derived for autocorrelation by Anselin (1995) in juxtaposition to Moran's I, a global autocorrelation metric. As with the LISA metric, S^3 values are generally additive, so that the total area under the logarithmic graph is a function of the overall sinuosity of the graphed feature.

A third benefit of the S^3 variant on Richardson's original plot is the ability to utilize distinctions between stream line features as graphed (i.e., by distinguishing among patterns of S^3) as one of several training variables that could be used for deep learning classification of stream features. Because the variant on the Richardson plot indicates specific ranges of stride length in which the feature lengths change, it provides a visual and quantitative tool to distinguish among features with high, medium or low proportions of large, intermediate and small meander bends. It may be possible to utilize this information in a deep learning analysis as one of several training layers to support classification of various geomorphologic characteristics. Further research using a larger sample set is required to demonstrate this empirically. To summarize, initial exploration indicates that the variant on the Richardson plot may provide a visual and quantitative tool to support the protection of sinuosity during generalization, and it may also provide a metric for geomorphic classification of river systems and landscapes.

4. Discussion and Implications

This paper describes a variant of the Richardson plot. The Richardson plot has been used widely to illustrate fractal dimension of naturally occurring landscape features that are understood to be sensitive to changes in scale or resolution, such as hydrographic features (coastlines, river channels, terrain, etc.). The original Richardson plot was foundational in Mandelbrot's (1967) derivation of fractal dimension, working from the slope of a linear regression line passing through a log-log point cloud relating stride length to feature length. The variant described here is based upon a metric that we name Scale-Specific Sinuosity (S^3) and defined as negative one (-1) times the slope of the Richardson plot for a given stride length. It can be interpreted as the rate at which feature length increases when measured at finer granularity (i.e., finer units of measure). A plot of S^3 against stride length offers a frequency distribution whose area under the curve reflects total sinuosity, and whose points mark the amount of sinuosity contributed to the total value at each stride length. The S^3 plot quantifies changes in fractal dimension at specific scales, and highlights resolutions at which a feature is particularly sensitive to slight changes in units of measurement.

Sinuosity has been a focus for many analysts who have derived a multitude of metrics to characterize the complexity of linear features, polygonal shapes and geometric solids. The S^3 metric quantifies systematic relations with fractal dimension and informs several important tasks that are specific to naturally occurring features. In addition to its utility for preserving sinuosity during cartographic generalization, the metric may also prove useful for characterizing or classifying linear features, especially streams with meander patterns that contain larger and smaller bends and convolutions. For example, it might be possible to quantify stream features by sinuosity magnitude and the distribution of sinuosity across scales based on proportions of area under the S^3 curve between specific ranges of stride lengths. This should be correlated with the number of meanders for specific meander sizes.

The S^3 metric can be readily applied to a broader set of naturally occurring features such as coastlines, boundaries of land cover or vegetation polygons, or animal migration tracks. Multi-scale representations and analyses of any of these types of features reasonably could be expected to have sinuosity constraints imposed on them. For example, land cover and vegetation frequently serve as ancillary variables in landscape modeling; and while neither carries the same level of scale-sensitivity as does hydrography or terrain, it might be very important in a modeling task to ensure that sinuosity is preserved within a specified range of values across scales to preserve logical relationships (e.g., landscape fragmentation) with modeled landscape values. One might also consider application of the S^3 metric to examine complexity of human-made features, as in the case of urban expansion, development and structure (Batty and Longley 1994).

This work may also have important analytical implications for surface modelling applications where resulting values might depend as much on the resolution as on the geomorphic process under investigation. For example, in natural hazards analysis such as flooding or debris flows following wildland fire, estimates of flow accumulation or of damage from landslides may depend upon the surface complexity of terrain. Derivation of scale-specific sinuosity for linear features leads naturally to consider whether the metric can also be applied to higher dimensionality features. Such consideration however can be documented only with further research.

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